REAL OPTION VALUE

CHAPTER 11 MULTI-FACTOR RENEWALS

Renewal models are usually based on the assumption that real assets such as equipment deteriorate with age and/or usage, and periodically have to be replaced with replica equipment. The models described in this chapter are based on various assumptions regarding the drifts and volatility of equipment inputs and outputs, the number of times equipment might be replaced, and the feasibility or opportunity for asset abandonment. There are two basic focus areas: the optimal timing of renewal, or abandonment; and the real option value of the renewal opportunity at any moment of time.

The most general renewal model is based on the assumption that expected sales decay and operating costs increase with the equipment age/usage, and both are variable. Typical assumptions are that both sales and costs follow a geometric Brownian motion, with a constant drift and volatility over time. There are an unlimited number of times equipment can be renewed or property renovated (and the production process or underlying economic world is perpetual), with a constant investment cost for renewal which will bring sales and costs back to the original levels.

Generally, higher sales volatility is associated with delayed renewals (lower current sales levels that justify a renewal). The correlation between sales and operating costs has a significant influence on the renewal boundary, with higher correlation associated with less delayed renewal (higher current sales levels that justify a renewal). Also, the sales expected immediately following an equipment renewal has a greater relative bearing on the spread between the current sales and costs that justifies a renewal than either its operating or renewal investment cost.

The multi-factor renewal problem is to find a $\hat{P}$ given $\hat{C}$ (the sales level P at which a renewal decision should be made, if the cost level equals $\hat{C}$), which is a solution to a small set of simultaneous equations. This solution method is computationally easy and transparent.

Other specific renewal models are based on limiting assumptions: the deterministic NPV models assume all inputs are constant; one-factor renewal models such as Dobbs (2004) assume either sales or costs are constant; single renewal models assume only one renewal is possible; and the abandonment model assumes no further renewals are possible, but exit is optional. There are easy analytical solutions for all of these specific renewal/abandonment models.

Asset renewal is relevant for many types of real assets. Major examples are: in transportation, where airplanes, buses, railcars, autos and taxis, cycles, ships (and containers) and shoes have limited life due to physical deterioration as well as to innovations in new equipment such as regarding fuel efficiency or speed; process equipment such as farm tractors, construction machines, office support systems for copying computing and communicating; stationary systems such as power stations, pipelines, and hotels, where periodic renovation is required to bring quality back to a level which justifies high revenues, and costs down to efficient operating levels; non-durable and semi-durable support devices such as software, medical implants, and clothes; finally, limited life consumables such as drugs, which companies attempt to replace with newer drugs as the old products come off patent. You can think of many more applications, including renewing real option texts and teachers.

Early deterministic replacement theory was developed by Faustmann in 1849 on optimal tree harvesting and replanting, see Linnard and Gane (1968). The optimal replacement policy for equipment is extended by Lutz and Lutz (1951) to the continuous time domain. The effects of depreciation and tax, salvage values and technology on replacement policy are covered in Merrett and Sykes (1966) and Bierman and Smidt (2007).

There are other stochastic renewal models, such as Dobbs (2004), who focuses on only stochastic costs.

When two driving factors in a real option model are both subject to variability, some authors such as Paxson and Pinto (2005) attempt to reduce the dimensions by similarity methods, or focus on net cash flows rather than the separate elements. Adkins and Paxson (2011) provide a quasi-analytical implicit solution to a two-factor real option renewal model without having to reduce the dimensions. As the practitioner focuses on the specific critical drivers of periodic equipment renewals, the current sales or cost levels and expected volatilities and correlation plus expected reversionary sales and costs upon renewal can be entered into quarterly or even monthly management accounts using spreadsheets in order to make appropriately timed renewal decisions.

11.1 MULTI-FACTOR MULTIPLE RENEWALS

Sales of goods produced by the asset at any time are denoted by P, operating costs by C, and the net cash flow is $P - C$. At installation, the sales and operating cost levels for the newly installed asset start at P_I and C_I, respectively. It is assumed that sales and operating costs change at the annualized continuous rate of $\alpha_P < 0$ and $\alpha_C > 0$, respectively, so generally $P < P_I$ and $C > C_I$. The renewal re-investment cost is denoted by the known constant K. The incumbent's residual salvage value at the renewal event is either zero, or it can be absorbed in K as long as it is a deterministic constant.

It is assumed the two uncertain variables follow distinct geometric Brownian motion processes with drift. For $X \in \{P, C\}$:

$$dX = \alpha_X X dt + \sigma_X X dz_X \tag{11.1}$$

where α_X is the instantaneous drift rate, σ_X is the instantaneous volatility rate, and dz_X is the increment of a standard Wiener process. Dependence between the two uncertain variables is described by the instantaneous covariance term $\rho\sigma_P\sigma_C$ where $Cov[dP, dC] = \rho\sigma_P\sigma_C PCdt$ and $|\rho| \leq 1$.

Suppose the renewal can be represented by the set $\{\hat{P}, \hat{C}\}$, where $\hat{P}$ and $\hat{C}$ denote the respective optimal threshold levels for sales and operating costs that signal renewal. Renewal is triggered when the prevailing operating cost and sales levels simultaneously attain their respective thresholds.

The function F is defined as the value of the incumbent asset including its embedded renewal option. All renewal decisions are treated as being made in isolation to any other enacted policies, so scale and other flexibilities are assumed to be absent. The value of F depends on the prevailing sales and operating cost levels so $F = F(P, C)$. By assuming complete markets, standard contingent claims analysis can be applied to the asset with value F to determine its risk neutral valuation relationship. This is expressed as the differential equation:

$$\begin{aligned} &\tfrac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F}{\partial P^2} + \tfrac{1}{2}\sigma_C^2 C^2 \frac{\partial^2 F}{\partial C^2} + \rho\sigma_P\sigma_C PC \frac{\partial^2 F}{\partial P \partial C} \\ &\qquad + \theta_P P \frac{\partial F}{\partial P} + \theta_C C \frac{\partial F}{\partial C} - rF + (P - C) = 0. \end{aligned} \tag{11.2}$$

where r is the risk-free rate of interest, and θ_P and θ_C are the risk-adjusted drift rates, respectively, for sales and operating costs. It is assumed that $r - \theta_X > 0$.

The simplest kind of generic function satisfying the homogenous part of (11.2) takes the form:

$$F_H(P, C) = AP^{\beta}C^{\eta} \tag{11.3}$$

where A is a parameter whose value has to be determined.

The functional form (11.3) satisfies the homogenous part of (11.2). Substituting (11.3) in the homogenous part of (11.2) reveals that the risk neutral valuation relationship is satisfied by the following characteristic root equation:

$$Q(\beta, \eta) = \tfrac{1}{2}\sigma_P^2 \beta(\beta - 1) + \tfrac{1}{2}\sigma_C^2 \eta(\eta - 1) + \rho\sigma_P\sigma_C \beta\eta + \theta_P\beta + \theta_C\eta - r = 0. \tag{11.4}$$

This is the two-factor equivalent of the β quadratic equation for the one-factor model in Chapter 4A.

By ignoring higher derivatives greater than one, the particular solution F_P to (11.2) is:

$$F_P(P,C) = \frac{P}{r-\theta_P} - \frac{C}{r-\theta_C}. \quad (11.5)$$

When sales approach infinity, there is no economic justification for renewing the asset, so the renewal option value tends to zero and F_P dominates the value of F. In contrast, a near zero value of P makes asset renewal inevitable, which is reflected in an infinitely large renewal option value with F_H dominating the value of F. Similarly, there is no economic justification for renewing the asset when operating costs are near zero, so the renewal option value will be dominated by the value of F_P. The asset should be renewed when C becomes infinitely large, when the renewal option value becomes exceedingly large and dominates the value of F_P. From these boundary conditions, β and η solutions for (11.4) are $\beta<0$, $\eta>0$.

Stitching together the particular and homogenous solutions, (11.5) and (11.3), produces the value of the asset and its renewal option:

$$F = A\,P^{\beta} C^{\eta} + \frac{P}{r-\theta_P} - \frac{C}{r-\theta_C} \quad (11.6)$$

The value matching boundary condition identifies the renewal event when P and C simultaneously attain their respective threshold levels $\hat{P}$ and $\hat{C}$. At the renewal event, the incumbent asset value including its renewal option is given by $F(\hat{P},\hat{C})$. After expending the renewal investment cost K, the incumbent is exchanged for a replica having an asset value including its renewal option $F(P_I,C_I)$. The value matching relationship, $F(\hat{P},\hat{C}) = F(P_I,C_I) - K$, can be expressed as:

$$A\,\hat{P}^{\beta}\hat{C}^{\eta} + \frac{\hat{P}}{r-\theta_P} - \frac{\hat{C}}{r-\theta_C} = A\,P_I^{\beta} C_I^{\eta} + \frac{P_I}{r-\theta_P} - \frac{C_I}{r-\theta_C} - K. \quad (11.7)$$

There are two associated smooth pasting conditions, one for each factor, so that:

$$A = -\frac{\hat{P}}{\beta\,(r-\theta_P)} \times \frac{1}{\hat{P}^{\beta}\hat{C}^{\eta}} = \frac{\hat{C}}{\eta\,(r-\theta_C)} \times \frac{1}{\hat{P}^{\beta}\,\hat{C}^{\eta}} \quad (11.8)$$

Clearly $A \geq 0$ as required, since $\beta < 0$ and $\eta > 0$.

Substituting A from (11.8) into (11.6) the valuation function is:

$$F = (\frac{P}{\hat{P}})^{\beta}(\frac{C}{\hat{C}})^{\eta}\frac{\hat{P}}{-\beta(r-\theta_P)} + \frac{P}{r-\theta_P} - \frac{C}{r-\theta_C} \tag{11.9}$$

By substituting (11.8) in $F(\hat{P},\hat{C})$ and recognizing that $F(P_I, C_I) - K$ must be positive otherwise no renewal investment would ever be made, the asset value including the renewal option $\hat{F}$ at the renewal event is:

$$\hat{F} = F(\hat{P},\hat{C}) = \frac{\hat{P}}{\beta\ (r-\theta_P)}(\beta\ +\eta\ -1) = \frac{\hat{C}}{\eta\ (r-\theta_C)}(1-\beta\ -\eta\ ) > 0. \tag{11.10}$$

This implies that $\beta\ +\eta\ < 1$.

From (11.8):

$$\frac{\hat{P}}{-\beta\ (r-\theta_P)} - \frac{\hat{C}}{\eta\ (r-\theta_C)} = 0 \tag{11.11}$$

provided $\hat{C} > 0$. Since (11.11) implies that $\hat{P} - \hat{C}$ can be negative, it is possible that renewal occurs for a negative prevailing net cash flow, which is a result that contrasts with the deterministic replacement model and demonstrates the existence of hysteresis.

Using (11.8) to eliminate A, the value matching relationship (11.7) can be expressed as:

$$\frac{\hat{P}}{r-\theta_P} - \frac{\hat{C}}{r-\theta_C} + \frac{\hat{C}}{\eta\ (r-\theta_C)}\left\{1 - \frac{P_I^{\beta} C_I^{\eta}}{\hat{P}^{\beta}\ \hat{C}^{\eta}}\right\} = \frac{P_I}{r-\theta_P} - \frac{C_I}{r-\theta_C} - K. \tag{11.12}$$

By substituting (11.11) in (11.12) to eliminate $\hat{P}$, this can be expressed as:

$$H(\beta\ ,\eta\ |\hat{C}) = \frac{\hat{C}}{\eta\ (r-\theta_C)}\left(1-\beta\ -\eta\ - \frac{P_I^{\beta}\ C_I^{\eta}}{\hat{C}^{\beta\ +\eta}}\left(\frac{-\beta\ (r-\theta_P)}{\eta\ (r-\theta_C)}\right)^{-\beta}\right) - \frac{P_I}{r-\theta_P} + \frac{C_I}{r-\theta_C} + K = 0. \tag{11.13}$$

The characteristic root equation (11.4), the reduced form value matching relationship (11.13) and the reduced form smooth pasting condition (11.11) constitute the two-factor renewal model from which the discriminatory boundary is generated. To determine the boundary, set these three equations equal to zero, by changing β, η and $\hat{P}$, corresponding to some assumed $\hat{C}$.

11.2 RESTRICTED RENEWALS

Suppose that there are a finite number of renewal opportunities, due to technical innovations, or equipment-type obsolescence, or simply supplier choice (what happened to the convenient old Word equation editor, or wooden wheels, or Nike Pegagus running shoes?). Williams (1997) introduced a replacement indexation style, so that the index J, J=0, 1, ..., denotes the number of remaining renewal opportunities. $F_J(P,C)$ denotes the incumbent asset value when J further renewal opportunities are available. For $J=0$, there are no future remaining renewal opportunities available, but the owner has an option to abandon the asset. For $J=1$, one further renewal opportunity remains, after which the only available opportunity is abandonment.

A. Abandonment

When there are no further renewals and $J=0$, the incumbent asset value including the abandonment option is denoted by $F_0(P,C)$. The valuation relationship satisfies the same PDE as (11.2) and the solution takes on a similar form as (11.7) except for the parameter changes:

$$F_0(P,C) = A_0 P^{\beta_0} C^{\eta_0} + \frac{P}{r-\theta_P} - \frac{C}{r-\theta_C}. \tag{11.14}$$

The value matching relationship for $J=0$ becomes:

$$F_0(\hat{P}_0,\hat{C}_0) = A_0 \hat{P}_0^{\beta_0} \hat{C}_0^{\eta_0} + \frac{\hat{P}_0}{r-\theta_P} - \frac{\hat{C}_0}{r-\theta_C} = 0. \tag{11.15}$$

Assuming no salvage value, $\beta_0 + \eta_0 = 1$, (11.16) so their solution values can be directly evaluated from (11.4). The renewal boundary is linear and given by:

$$\frac{\hat{P}_0}{-\beta_0\left(r-\theta_P\right)}-\frac{\hat{C}_0}{\eta_0\left(r-\theta_C\right)}=0. \tag{11.17}$$

B. Single Renewal Option

When there is only one remaining renewal opportunity so $J=1$, the solution is derived directly from the model with multiple opportunities by eliminating the renewal option from the replica asset value. Using the subscript s to denote the single renewal opportunity, then from (11.8), the value matching relationship becomes:

$$A_s\hat{P}_s^{\beta_s}\hat{C}_s^{\eta_s}+\frac{\hat{P}_s}{r-\theta_P}-\frac{\hat{C}_s}{r-\theta_C}=\frac{P_I}{r-\theta_P}-\frac{C_I}{r-\theta_C}-K\ . \tag{11.18}$$

It follows that the two smooth pasting conditions associated with (11.18) imply (11.9), and by substituting and rearranging, the reduced value matching condition is:

$$\frac{\hat{P}_s}{r-\theta_P}\left[\frac{\beta_s-1}{\beta_s}\right]-\frac{\hat{C}_s}{r-\theta_C}-\frac{P_I}{r-\theta_P}+\frac{C_I}{r-\theta_C}+K=0\,. \tag{11.19}$$

A smooth pasting condition is:

$$\frac{\hat{P}_s}{-\beta_s\left(r-\theta_P\right)}-\frac{\hat{C}_s}{\eta_s\left(r-\theta_C\right)}=0. \tag{11.20}$$

The single renewal boundary is evaluated by solving the three simultaneous equations: the reduced form value matching relationship (11.19), the reduced form smooth pasting condition (11.20) and the characteristic root equation (11.4). A simpler version of (11.19) is found by using (11.20) to eliminate $\hat{P}$:

$$\frac{\hat{C}_s}{\eta_s\left(r-\theta_C\right)}\left[1-\beta_s-\eta_s\right]-\frac{P_I}{r-\theta_P}+\frac{C_I}{r-\theta_C}+K=0\,. \qquad (11.21)$$

The renewal boundaries for the three cases of an infinite number of renewal opportunities, a single renewal opportunity and the abandonment opportunity are vertically stacked: the boundary for the infinite renewal model entirely lies above that for the single renewal model, which entirely lies above that for the abandonment model, that is for every operating cost threshold level, $\hat{\mathrm{P}}_\infty > \hat{\mathrm{P}}_\mathrm{s} > \hat{\mathrm{P}}_0$. This means that the trajectory of prevailing sales and operating cost levels, starting from their respective initial levels $\mathrm{P_I}$ and $\mathrm{C_I}$ at renewal, will always hit the infinite renewal boundary first before reaching either the single renewal or the abandonment boundaries.

Provided that $\mathrm{P_I}$ and $\mathrm{C_I}$ remain unaltered during the project lifetime, the infinite renewal policy always dominates the other two policies. The dominance of the infinite renewal policy is overruled whenever there is an appropriate decline in the initial sales level, or an appropriate increase in either the initial operating cost or the re-investment cost. Suitable changes in any of these three will bring about a switch away from the infinite renewal to the abandonment policy. If the abandonment opportunity is to become viable at the renewal event for some initial sales level, then both the incumbent asset value and the replica asset value less the re-investment cost have to equal zero, which is the abandonment value:

C. Deterministic and One-Factor Renewals

Adkins and Paxson (2011) show that the two factor deterministic renewal model is a special case of the two factor stochastic model. Using the suffix * to denote the optimal deterministic value, the first order condition for the maximum NPV for an infinite chain of replica assets with a constant renewal interval T* simplifies to:

$$\hat{P}\left(\frac{1}{r}+\frac{\theta_P}{r}\times\frac{e^{-r\hat{T}}}{r-\theta_P}\right)-\hat{C}\left(\frac{1}{r}+\frac{\theta_C}{r}\times\frac{e^{-r\hat{T}}}{r-\theta_C}\right)=\frac{P_I}{r-\theta_P}-\frac{C_I}{r-\theta_C}-K$$

(11.22)

The parameters of the real option renewal model are amended by setting $\sigma_P=\sigma_C=0$, then from (11.4) and (11.11), and using the subscript d to denote the deterministic version of the general renewal model, $T^*=\hat{T}$, where the optimal cycle time is:

$$\hat{T}=\frac{1}{\theta_C}\ln\left(\frac{\hat{C}_d}{C_I}\right)=\frac{1}{\theta_P}\ln\left(\frac{\hat{P}_d}{P_I}\right) \tag{11.23}$$

$$\theta_P\beta_d+\theta_C\eta_d-r=0, \tag{11.24}$$

$$\left(\frac{P_I}{\hat{P}_d}\right)^{\beta}\left(\frac{C_I}{\hat{C}_d}\right)^{\eta}-e^{-r\hat{T}}=0 \tag{11.25}$$

Converting the Dobbs (2004) one-factor (cost) model to a one-factor model with only uncertain sales, then $\theta_C=0$, $\sigma_C=0$ and $\eta=0$ in the two factor stochastic model. The sales threshold level is derived from (11.4), (11.11) and (11.13):

$$\frac{\hat{P}_1}{\beta_1(r-\theta_P)}\left(\beta_1-1+\left(\frac{P_I}{\hat{P}_1}\right)^{\beta_1}\right)-\frac{P_I}{r-\theta_P}+K=0, \tag{11.26}$$

$$\beta_1=\left(\tfrac{1}{2}-\frac{\theta_P}{\sigma_P^2}\right)-\sqrt{\left(\tfrac{1}{2}-\frac{\theta_P}{\sigma_P^2}\right)^2+\frac{2r}{\sigma_P^2}}. \tag{11.27}$$

D. Deterministic Technological Progress

Thus far it has been assumed that P reverts to P_I and C to C_I (repeatedly in the multiple case), when equipment is renewed, or property is renovated. Both competitive forces and the threat of new technology are likely to motivate asset suppliers to continuously improve product performance. If these improvements are realized through continuous changes in the initial attributes, then over a period of time, we would observe falls in either the initial operating cost level or re-investment cost for the succeeding asset, or increases in its initial revenue level. Suppose anticipated technological progress is determined through a time dependent initial operating cost level (like Moore's law for Intel processing power). For the succeeding asset, the new initial operating cost level, which is denoted by C_N, can be expressed by a growth function with a continuous constant rate θ_N, that is $\mathrm{d}C_N = \theta_N C_N \mathrm{d}t$. This growth parameter is expected to be negative, since performance improvements are presumed to be embedded in the succeeding asset with C_N declining over time. The presence of a new initial operating cost level in the model means that the value function, which is denoted by F_3, depends on three factors, the new initial operating cost level as well as the prevailing levels for the revenues and operating cost. In the two-factor model, the replacement option value is expressed as a product power function of the two factors, revenues and operating costs. For the three-factor model under consideration, a product power function can be adopted but now of three factors, revenues, operating costs and new initial operating cost level, to represent the replacement option value. So, the valuation function becomes:

$$F_3\left(P,C,C_N\right) = A_3 P^{\beta_3} C^{\eta_3} C_N^{\gamma_3} + \frac{P}{r-\theta_P} - \frac{C}{r-\theta_C}, \qquad (11.28)$$

where $A_3 P^{\beta_3} C^{\eta_3} C_N^{\gamma_3}$, with $A_3 > 0$, represents the option value with power parameters β_3, η_3 and γ_3. Again, the term $P/\left(r-\theta_P\right) - C/\left(r-\theta_C\right)$ denotes the asset value in the absence of any optionality. Since a stronger economic incentive exists for replacing the incumbent when the initial operating cost level is low rather than high, we would expect the replacement option to increase in value as C_N decreases, so we conjecture that the value of γ_3 should be negative. The replacement event is signalled when the three factor levels, P, C and C_N, simultaneously attain their respective optimal threshold levels, $\hat{P}_3$, $\hat{C}_3$ and $\hat{C}_{3N}$. Collectively, these three optimal thresholds form the timing boundary, which is determined from the model solution, made up of the economic conditions signalling an optimal replacement, that is the value matching relationship and the smooth pasting conditions, plus the characteristic root equation.

Because value is conserved at replacement, the incumbent value $F_3\left(\hat{P}_3, \hat{C}_3, \hat{C}_{3N}\right)$ has to exactly balance the succeeding asset value $F_3\left(P_I, \hat{C}_N, \hat{C}_N\right)$, less the re-investment cost K. By using (11.28), the value matching relationship can be expressed as:

$$A_3 \hat{P}_3^{\beta_3} \hat{C}_3^{\eta_3} \hat{C}_{3N}^{\gamma_3} + \frac{\hat{P}_3}{r-\theta_P} - \frac{\hat{C}_3}{r-\theta_C} = A_3 P_I^{\beta_3} \hat{C}_{3N}^{\eta_3+\gamma_3} + \frac{P_I}{r-\theta_P} - \frac{\hat{C}_{3N}}{r-\theta_C} - K \,. \tag{11.29}$$

Replacement is optimal whenever the smooth pasting conditions are obtained. Associated with (11.29), there are three smooth pasting conditions, for P, C and C_N, respectively, which can be expressed as:

$$\beta_3 A_3 \hat{P}_3^{\beta_3} \hat{C}_3^{\eta_3} \hat{C}_{3N}^{\gamma_3} + \frac{\hat{P}_3}{r-\theta_P} = 0 \,, \tag{11.30}$$

$$\eta_3 A_3 \hat{P}_3^{\beta_3} \hat{C}_3^{\eta_3} \hat{C}_{3N}^{\gamma_3} - \frac{\hat{C}_3}{r-\theta_C} = 0 \,, \tag{11.31}$$

$$\gamma_3 A_3 \hat{P}_3^{\beta_3} \hat{C}_3^{\eta_3} \hat{C}_{3N}^{\gamma_3} = \left(\eta_3 + \gamma_3\right) A_3 P_I^{\beta_3} \hat{C}_{3N}^{\eta_3+\gamma_3} - \frac{\hat{C}_{3N}}{r-\theta_C} \,. \tag{11.32}$$

We observe that $\beta_3 < 0$ and $\eta_3 > 0$. Also, since $P_I^{\beta_3} \hat{C}_{3N}^{\eta_3} < \hat{P}_3^{\beta_3} \hat{C}_3^{\eta_3}$ because $\hat{P}_3 < P_I$ and $\hat{C}_3 > \hat{C}_{3N}$, then from (11.32) $\gamma_3 < 0$.

From (11.30) and (11.31), then:

$$\frac{\hat{P}_3}{\beta_3\left(r-\theta_P\right)} + \frac{\hat{C}_3}{\eta_3\left(r-\theta_C\right)} = 0 \,. \tag{11.33}$$

By combining (11.31) and (11.32), A_3 can be eliminated from (11.29) to yield:

$$\frac{\hat{C}_3 - \hat{C}_{3N}}{r-\theta_C} = \frac{\eta_3+\gamma_3}{\eta_3+\gamma_3-1}\left[K - \frac{P_I - \hat{P}_3}{r-\theta_P}\right]. \tag{11.34}$$

An optimal replacement is justified when the operating cost value improvement $\left(\hat{C}_3 - \hat{C}_{3N}\right)/\left(r - \theta_C\right)$ equals the re-investment cost less the revenue value improvement $\left(P_I - \hat{P}_3\right)/\left(r - \theta_P\right)$, adjusted by the mark-up factor.

Also, A_3 can be eliminated from (11.29) by using (11.31) to yield:

$$\frac{P_I - \hat{P}_3}{r - \theta_P} + \frac{\hat{C}_3 - \hat{C}_{3N}}{r - \theta_C} = K + \frac{\hat{C}_3}{\eta_3\left(r - \theta_C\right)}\left[1 - \frac{P_I^{\beta_3}}{\hat{P}_3^{\beta_3}}\frac{\hat{C}_{3N}^{\eta_3}}{\hat{C}_3^{\eta_3}}\right]. \qquad (11.35)$$

Since $P_I^{\beta_3}\hat{C}_{3N}^{\eta_3} < \hat{P}_3^{\beta_3}\hat{C}_3^{\eta_3}$, replacement is optimal whenever the sum of the value improvements rendered by the replacement exceeds the re-investment cost.

The final component of the model is the characteristic root equation:

$$\frac{1}{2}\sigma_P^2\beta_3(\beta_3 - 1) + \frac{1}{2}\sigma_C^2\eta_3(\eta_3 - 1) + \rho\sigma_P\sigma_C\beta_3\eta_3 + \theta_P\beta_3 + \eta_3\theta_C + \gamma_3\theta_N - r = 0\,. \qquad (11.36)$$

There are four equations for the technological progress model. These are (i) the reduced form smooth pasting condition, (11.33), (ii) and (iii) two reduced form value matching relationships, (11.34) and (11.35), and (iv) the characteristic root equation, (11.36).

The real replacement option value at current P_3, C_3 and C_{3N}, ROV_3, is obtained by solving (11.30) for A_3, and substituting in the first part of (11.28):

$$ROV_3 = \frac{\hat{P}_3}{-\beta_3(r - \theta_P)}\left[\frac{P_3^{\beta_3}}{\hat{P}_3^{\beta_3}}\frac{C_{3N}^{\gamma_3}}{\hat{C}_{3N}^{\gamma_3}}\frac{C_3^{\eta_3}}{\hat{C}_3^{\eta_3}}\right] \qquad (11.37)$$

11.3 SOLVING SETS OF EQUATIONS TO FIND $\hat{P}$

The optimal renewal boundary is determined for a representative range of $\hat{P}$ and $\hat{C}$ using the base case data and spreadsheets in Figures 11.1, 11.2 and 11.3. In order to compare the general two stochastic factor case with the conventional deterministic case, first the deterministic results are calculated in Figure 11.1. Simultaneously solving equations 11.23-11.24-11.25 produces the results that ignoring sales and cost volatility justifies renewing equipment when $\hat{P}$ falls to 65 if $\hat{C}$ has increased to almost 30. The NPV of this renewal decision is zero at r=7%, when the P drift is -2% and C drift is 4%.

Figure 11.1

	A	B	C	D	E	F	G	H	I	J	K	L
1		**Renewal Template**										
2	INPUT	**Deterministic**										
3	P_I	80.00										
4	C_I	20.00										
5	K	100.00										
6	C*	29.95										
7	σ_P	0.00										
8	σ_C	0.00										
9	ρ	0.00										
10	r	0.07										
11	θ_P	-0.02										
12	θ_C	0.04										
13												
14	OUTPUT											
15	$Q(\beta,\eta)$	0.0000										
16	SP	0.0000										
17	SUM	0.0000										
18	β	-0.0451										
19	η	1.7274										
20	P*	**65.371**										
21	C*	29.953										
22	T^*_C	10.098										
23	T^*_P	10.098										
24	SOLVER	0.000										
25	P*-C*	35.418										
26	Deterministic											
27	$Q(\beta,\eta)$	B11*B18+B12*B19-B10				EQ 24						
28	SP	((B3/B20)^B18)*((B4/B21)^B19)-EXP(-B10*B22)				EQ 25						
29	SOLVER	SET B24=0,CHANGING B18:B21,B22=B23										
30												
31	T^*_C	(1/B12)*(LN(B21/B4))				EQ 23						
32	T^*_P	(1/B11)*(LN(B20/B3))				EQ 23						
33												
34	P* VALUE	831.52										
35	C*VALUE	709.29										
36	Renewal V-K	122.22										
37	NPV=0	0.0000										
38												
39	P*	B20*((1/B10)+(B11/B10)*(EXP(-B10*B22)/(B10-B11)))										
40	C*	B21*((1/B10)+(B12/B10)*(EXP(-B10*B22)/(B10-B12)))										
41	Renewal V-K	B3/(B10-B11)-B4/(B10-B12)-B5										
42	NPV=0	B34-B35-B36				EQ 22						
43												
44			ASSET DETERIORATION OVER THE YEARS									
45	YEARS	1	2	3	4	5	6	7	8	9	10	11
46	P	78.42	76.86	75.34	73.85	72.39	70.95	69.55	68.17	66.82	**65.50**	64.20
47	C	20.82	21.67	22.55	23.47	24.43	25.42	26.46	27.54	28.67	**29.84**	31.05
48	P-C	57.60	55.20	52.79	50.38	47.96	45.53	43.09	40.63	38.16	**35.66**	33.15
49	P	\$B\$3*EXP(\$B\$11*B45)										
50	C	\$B\$4*EXP(\$B\$12*B45)										

Note that the optimal renewal time is slightly over ten years. Assuming deterioration occurs at the end of the year, the rows 46 and 47 show that P would have declined to 65.50 and C increased to 29.84 at the end of the tenth year, so the trigger spread justifying a renewal is almost reached. The $\hat{P}$ VALUE and $\hat{C}$ VALUE are the first and second terms of (11.22, LHS), while the Renewal V-K is (11.22, RHS), the net present value at the reversion P and C less the renewal cost.

The deterministic method justifies a renewal when the $\hat{P}$ and $\hat{C}$ values at T* less the reversion P_I and C_I values less the renewal investment cost is zero.

Using $\hat{C}$ =29.95 for the two factor stochastic case, solving equations 11.4, 11.11 and 11.13 simultaneously, Figure 11.2 shows that a renewal would be justified only if P<57. If current P=60 and current C=30, a renewal would be justified only in the deterministic case. For comparison, the general renewal model setting $\sigma_P=\sigma_C=0$ replicates the deterministic result.

Figure 11.2

	A	B	C	D	E	F	G
1		**Renewal Template**					
2	INPUT	**Stochastic P & C**		**Stochastic Model P & C**			
3	P_I	80.00		80.00			
4	C_I	20.00		20.00			
5	K	100.00		100.00			
6	C*	29.95		29.95			
7	σ_P	**0.30**		**0.00**			
8	σ_C	**0.30**		**0.00**			
9	ρ	0.00		0.00			
10	r	0.07		0.07			
11	θ_P	-0.02		-0.02			
12	θ_C	0.04		0.04			
13							
14	OUTPUT						
15	Q(β,η)	0.0000		0.0000			
16	SP	0.0000		0.0000			
17	H(β,η)	0.0000		0.0000			
18	PART 1	1241.82		778.06			
19	PART 2	0.0984		0.1571			
20	PART 3	-122.22		-122.22			
21	SUM	0.0000		0.0000			
22							
23	β	-0.5107		-0.9335			
24	η	0.8040		1.2832			
25	P*	**57.076**		**65.371**			
26	P*-C*	27.123		35.418			
27							
28	Q(β,η)	0.5*(B7^2)*B23*(B23-1)+0.5*(B8^2)*B24*(B24-1)+B9*B7*B8*B23*B24+B11*B23+B12*B24-B10					EQ 4
29	SP	B25/(-B23*(B10-B11))-B6/(B24*(B10-B12))					EQ 11
30	H(β,η)	B18*B19+B20					EQ 13
31	PART 1	B6/(B24*(B10-B12))					
32	PART 2	1-B23-B24-((B3^B23)*(B4^B24)/(B6^(B23+B24)))*((-B23*(B10-B11)/(B24*(B10-B12)))^-B23)					
33	PART 3	-B3/(B10-B11)+B4/(B10-B12)+B5					
34							
35	SOLVER	SET B21=0,CHANGING B23:B25.					

The case of one stochastic factor, conveniently supposed to be P in this comparison, is calculated by solving equations (11.26) and (11.27) in Figure 11.3. Ignoring the cost drift of 4% used in the two factor and deterministic models, P would have to fall to below 52 before a renewal is justified. For comparison, setting $\theta_C = \sigma_C = 0$ in the stochastic two factor model and $C_I = 20$ replicates the one factor model results.

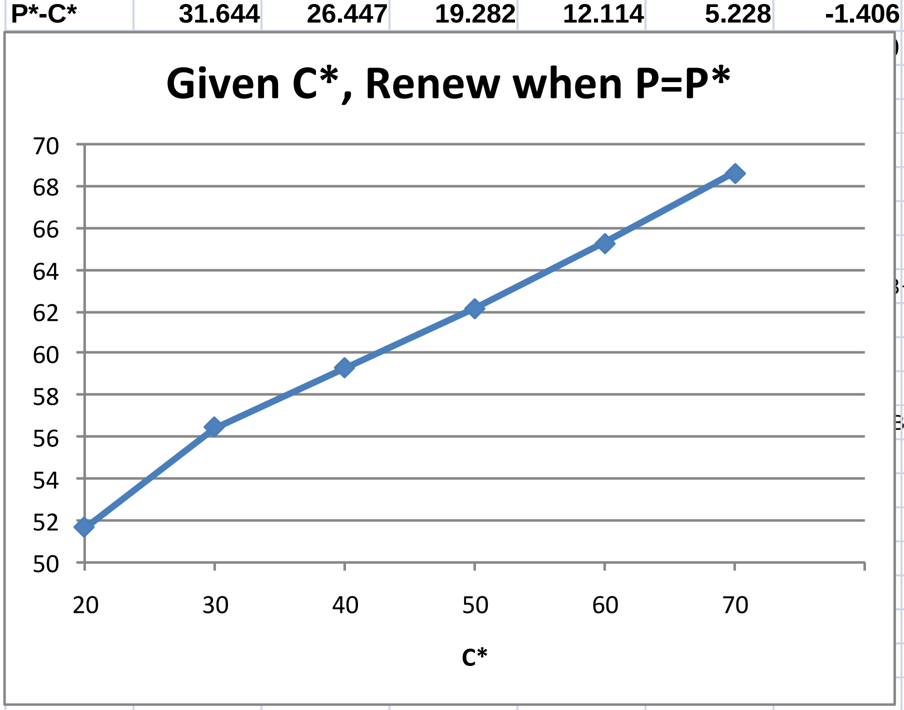
Figure 11.3

	A	B	C	D	E	F
1	**Renewal Template**					
2	INPUT	**Stochastic P**			**Stochastic P & C**	
3	P_I	80.00			80.00	
4	C_I	20.00			20.00	
5	K	100.00			100.00	
6	C*	20.00			20.00	
7	σ_P	0.30			**0.30**	
8	σ_C	0.00			**0.00**	
9	ρ	0.00			0.00	
10	r	0.07			0.07	
11	θ_P	-0.02			-0.02	
12	θ_C	0.00			**0.00**	
13						
14	OUTPUT					
15	$Q(\beta,\eta=0)$	0.0000		$Q(\beta,\eta)$	0.0000	
16	SP	0.0000		SP	0.0000	
17	SUM	0.0000		$H(\beta,\eta)$	0.0000	
18				SUM	0.0000	
19	β_1	-0.7190		β	-0.7190	
20	P*	**51.589**		P*	**51.589**	
21	η	0.0000		η	0.3584	
22	P*-C*	31.589		P*-C*	31.589	
23	Stochastic P					
24	$Q(\beta,\eta=0)$	(0.5-B11/(B7^2))-SQRT((0.5-B11/(B7^2))^2+2*B10/(B7^2))-B19				EQ 27
25	SP	(B20/(B19*(B10-B11)))*(B19-1+((B3/B20)^B19))-B3/(B10-B11)+B5				EQ 26
26						
27	SOLVER	SET B17=0, CHANGING B19:B20.				

It is optimal to renew assets whenever the prevailing sales and operating cost values are in the renewal region, and to continue with the incumbent if otherwise. The renewal boundary shown in Figure 11.4 has a positive but changing slope and therefore the relationship between $\hat{P}$ and $\hat{C}$ is not exactly proportionate.

Figure 11.4

American Multi-factor Perpetual Renewal Option							
INPUT	Stochastic P & C						
P_I	80.00	80.00	80.00	80.00	80.00	80.00	
C_I	20.00	20.00	20.00	20.00	20.00	20.00	
K	100.00	100.00	100.00	100.00	100.00	100.00	
C*	**20.00**	**30.00**	**40.00**	**50.00**	**60.00**	**70.00**	
σP	0.30	0.30	0.30	0.30	0.30	0.30	
σC	0.30	0.30	0.30	0.30	0.30	0.30	
ρ	0.00	0.00	0.00	0.00	0.00	0.00	
r	0.07	0.07	0.07	0.07	0.07	0.07	
θp	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	
θc	0.04	0.04	0.04	0.04	0.04	0.04	
OUTPUT							
Q(β,η)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 4
SP	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 11
H(β,η)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 13
SUM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
β4	-0.5805	-0.5075	-0.4456	-0.3988	-0.3640	-0.3377	
η4	0.6745	0.8092	0.9020	0.9631	1.0044	1.0338	
P*	**51.644**	**56.447**	**59.282**	**62.114**	**65.228**	**68.594**	
P*-C*	**31.644**	**26.447**	**19.282**	**12.114**	**5.228**	**-1.406**	



+B12*B24-B10

12)))^-B23)

P*=a+bC*		56.285	59.309	62.333	65.357	68.381
a		47.2121				
b		0.3024				
RSQ		0.9985				
Error Linear Regression		0.162	-0.027	-0.219	-0.129	0.213

The relationship implies that a positive trade-off exists between the two factors, and that an operating cost increase can be compensated by a relatively smaller sales increase before triggering a renewal event. Figure 11.4 indicates guidelines that management can use for deciding whether the asset should be renewed or not. If the discriminatory boundary in Figure 11.4 is sufficiently linear for $C \geq 30$, then the boundary could be represented by its least squares line and a simpler renewal decision rule would result. For these parameter values, an OLS regression $\hat{P} = a + b\hat{C}$ provides an approximate guideline.

Figure 11.5

	A	B	C	D	E	F	G	H
35				F= ROV + NPV				
36	P	55.00	60.00	65.00	70.00	75.00	80.00	
37	C	29.95	29.95	29.95	29.95	29.95	29.95	
38	P-C	25.05	30.05	35.05	40.05	45.05	50.05	
39	F	877.554	878.760	885.831	898.230	915.050	935.585	EQ 7
40	ROV	1265.533	1210.530	1162.045	1118.889	1080.153	1045.132	
41	NPV	-387.325	-331.770	-276.214	-220.659	-165.103	-109.548	
42	F*	877.554	877.554	877.554	877.554	877.554	877.554	EQ 10
43	A4	636.751	636.751	636.751	636.751	636.751	636.751	EQ 9
44								
45	F	IF(B36<B25,B42,B40+B41)						
46	ROV	B43*(B36^B23)*(B37^B24)						
47	NPV	B36/(B10-B11)-B37/(B10-B12)						
48	F*	(B25/(B23*(B10-B11)))*(B23+B24-1)						
49	A4	(-B25/(B23*(B10-B11)))*(1/((B25^B23)*(B6^B24)))						

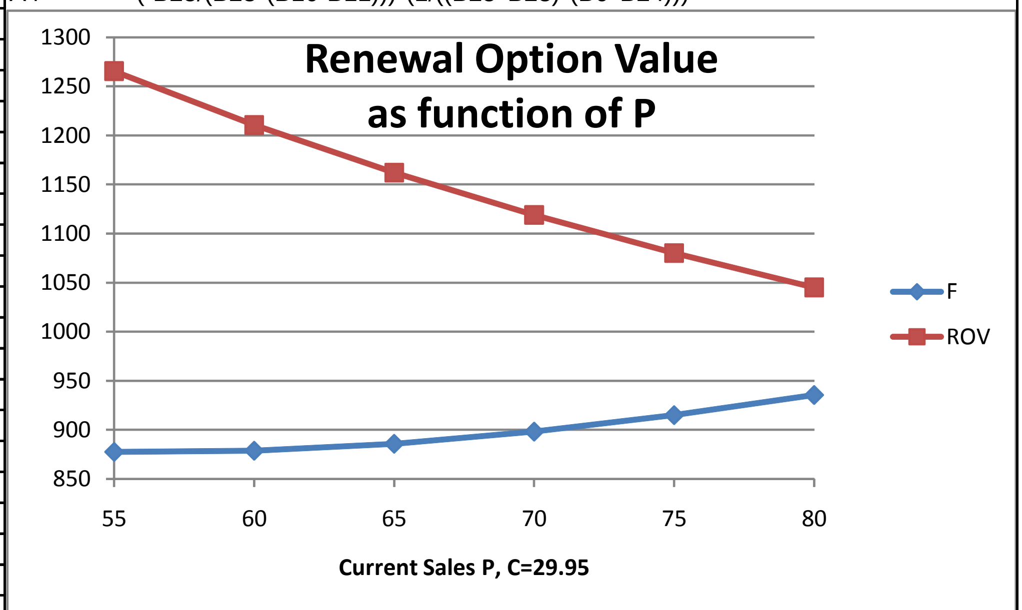


Figure 11.5 (which is an extension of Figure 11.2) shows F (EQS 11.7 and 11.9), the value of an on-going asset with an embedded renewal option (assuming the current parameter values), as a function of different values of current P. ROV is the "pure renewal option value" and NPV is the on-going value of F without a renewal option. In a world where the model assumptions are valid, F would be an appropriate substitute for the depreciated historical accounting figure for capital equipment, in real option balance sheets.

Figure 11.6 shows the triggers for a single remaining renewal opportunity across a range of C. In each case, the P trigger is much lower than for multiple renewals shown in Figure 11.4, so the issue of multiple versus single (or limited number of) possible renewals is a critical consideration in renewal decisions.

Figure 11.6

	A	B	C	D	E	F	G	H
1	**American Multi-factor Single Renewal Option**							
2	INPUT	Stochastic P & C						
3	P_I	80.00	80.00	80.00	80.00	80.00	80.00	
4	C_I	20.00	20.00	20.00	20.00	20.00	20.00	
5	K	100.00	100.00	100.00	100.00	100.00	100.00	
6	**C***	**20.00**	**30.00**	**40.00**	**50.00**	**60.00**	**70.00**	
7	σ_P	0.30	0.30	0.30	0.30	0.30	0.30	
8	σ_C	0.30	0.30	0.30	0.30	0.30	0.30	
9	ρ	0.00	0.00	0.00	0.00	0.00	0.00	
10	r	0.07	0.07	0.07	0.07	0.07	0.07	
11	θ_P	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	
12	θ_C	0.04	0.04	0.04	0.04	0.04	0.04	
13								
14	OUTPUT							
15	$Q(\beta,\eta)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 4
16	SP	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 20
17	VM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 21
18	SUM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19								
20	$\beta 14$	-0.2861	-0.2539	-0.2367	-0.2261	-0.2188	-0.2135	
21	$\eta 14$	1.0868	1.1173	1.1329	1.1423	1.1486	1.1531	
22	**P***	**15.7941**	**20.4522**	**25.0760**	**29.6854**	**34.2872**	**38.8847**	
23								
24	$\beta 14+\eta 14$	0.801	0.863	0.896	0.916	0.930	0.940	
25	$Q(\beta,\eta)$	0.5*(B7^2)*B20*(B20-1)+0.5*(B8^2)*B21*(B21-1)+B9*B7*B8*B20*B21+B11*B20+B12*B21-B10						
26	SP:	B22/(-B20*(B10-B11))-B6/(B21*(B10-B12))						
27	VM	(B6/(B21*(B10-B12)))*(1-B20-B21)-(B3/(B10-B11))+(B4/(B10-B12))+B5						
28	SOLVER	SET H18=0,CHANGING B20:G22.						

Figure 11.7 shows the abandonment trigger across a range of C. Note that P* is an exact linear (.4586) function of $\hat{C}$. In each case, the P trigger is less than for the single renewal opportunity. So the abandonment option is unlikely to be exercised even if reversionary C approaches 80, unless P has fallen to way below the current reversionary operating cost.

Figure 11.7

	A	B	C	D	E	F	G	H
1	**American Multi-factor Abandon Option**							
2	INPUT	Stochastic P & C						
3	P_I	80.00	80.00	80.00	80.00	80.00	80.00	
4	C_I	20.00	20.00	20.00	20.00	20.00	20.00	
5	K	100.00	100.00	100.00	100.00	100.00	100.00	
6	**C***	**20.00**	**30.00**	**40.00**	**50.00**	**60.00**	**70.00**	
7	σ_P	0.30	0.30	0.30	0.30	0.30	0.30	
8	σ_C	0.30	0.30	0.30	0.30	0.30	0.30	
9	ρ	0.00	0.00	0.00	0.00	0.00	0.00	
10	r	0.07	0.07	0.07	0.07	0.07	0.07	
11	θ_P	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	
12	θ_C	0.04	0.04	0.04	0.04	0.04	0.04	
13								
14	OUTPUT							
15	$Q(\beta,\eta)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 4
16	SP	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 17
17	$\beta_{04}+\eta_{04}-1$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	EQ 16
18	SUM	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	a	0.4586						
20	β_{04}	-0.1805	-0.1805	-0.1805	-0.1805	-0.1805	-0.1805	
21	η_{04}	1.1805	1.1805	1.1805	1.1805	1.1805	1.1805	
22	**P***	**9.1724**	**13.7586**	**18.3447**	**22.9309**	**27.5171**	**32.1033**	
23	**P*=aC***	9.1724	13.7586	18.3447	22.9309	27.5171	32.1033	
24								
25	$Q(\beta,\eta)$	0.5*(B7^2)*B20*(B20-1)+0.5*(B8^2)*B21*(B21-1)+B9*B7*B8*B20*B21+B11*B20+B12*B21-B10						
26	SP	B22/(-B20*(B10-B11))-B6/(B21*(B10-B12))						
27	a	(-B20/B21)*(B10-B11)/(B10-B12)						
28	**P*=aC***	B19*B6						
29	SOLVER	SET H18=0,CHANGING B20:G22.						

Figure 11.8 shows the effect on P* of considering technological progress, in this case assuming that θ_N the technological improvement in the reversionary C is 5% per annum.

Figure 11.8

	A	B	C	D
1	**EQUIPMENT RENEWAL: ANTICIPATED TECHNOLOGICAL PROGRESS**			
2	INPUT			
3	P_I	80.00		PI=P0
4	C_N	15.00		CN<C0
5	K	100.00		
6	C*	**29.95000**		
7	σ_P	0.30		
8	σ_C	0.30		
9	ρ	0.00		
10	r	0.070		
11	θ_P	**-0.02**		
12	θ_C	**0.04**		
13	θ_N	**-0.05**		
14	P	75.00		
15	C	30.00		
16	C_N	15.00		
17	$Q(\beta,\eta)$	0.0000	EQ 11.36	0.5*(B7^2)*B29*(B29-1)+0.5*(B8^2)*B30*(B30-1)+B9*B7*B8*B29*B30+B11*B29+B12*B30+B13*B31-B10
18	SP1	0.0000	EQ 11.33	B32/(B29*(B10-B11))+B6/(B30*(B10-B12))
19	VM2	0.0000	EQ 11.34	B25-B26*B27
20	VM1	0.0000	EQ 11.35	B21*B22-B23
21	VM1PART 1	1261.530		B6/(B30*(B10-B12))
22	PART 2	0.509		(1-((B3^B29)/(B32^B29))*((B33^B30)/(B6^B30)))
23	PART 3	641.958		((B3-B32)/(B10-B11)+(B6-B33)/(B10-B12)-B5)
24	VM2	0.000		B19
25	VM2 PART 1	498.333		(B6-B33)/(B10-B12)
26	PART 2	-3.470		((B30+B31)/(B30+B31-1))
27	PART 3	-143.624		(B5-((B3-B32)/(B10-B11)))
28	SOLVER SUM	0.0000000652		
29	β_4	-0.511		
30	η_4	0.791		
31	γ_4	-0.015		
32	P*	**58.074**		
33	C*N	**15.000**		
34	P*-C*	28.124		
35	F	**941.624**		IF(B14>B32,B36,B39)
36	F	941.624	EQ 11.28	B37+B38
37	ROV	**1108.290**	EQ 11.37	((B16/B33)^B31)*((B15/B6)^B30)*((B14/B32)^B29)*(B32/(-B29*(B10-B11)))
38	NPV	-166.667		(B14/(B10-B11))-B15/(B10-B12)
39	This figure depicts the boundary set (P*,C*) that indicates when P=<P* and			
40	C=>C*, asset renewal is justified, setting Eqs 11.33-11.36 =0, and assuming			
41	P(I)=80, C(0)>C(N), K=100, σP=.30, σC=.30, ρ=0, r=.07, θP=-.02, θC=.04, θN=-.05.			
42	SOLVER SUM CHANGING B29:B32			

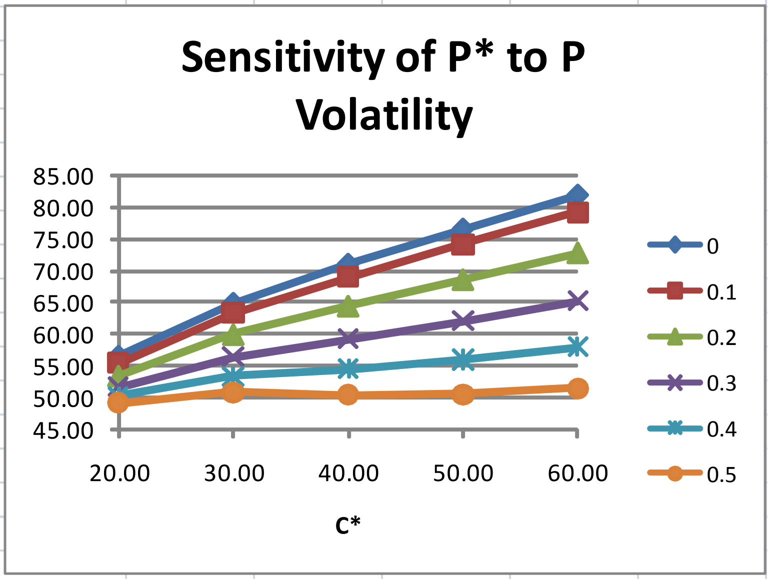
If the deterministic technological progress is consistent with C_N=15 when the renewal occurs, P*=58.07, slightly higher than when C_I=20.

11.4 WHAT INPUTS MATTER?

We now investigate the parametric effects on the optimal renewal boundary using the base case data in Figure 11.4. Figure 11.9 shows the effects of sales volatility σ_P on the sales threshold $\hat{P}$. It is apparent (assuming correlation equals 0) that increases in expected P volatility significantly reduce $\hat{P}$. The P zero volatility case (where, however, the cost is still considered stochastic) shows that for high $\hat{C}$, the $\hat{P}$ is always higher than for the two factor stochastic case. Another interpretation is that the expected subsequently repeated spread between expected pairs of $\hat{P}$ and $\hat{C}$ that justifies immediate renewal declines with increases in P volatility, and also with increases in $\hat{C}$. A similar pattern of effects is obtained for the operating cost volatility σ_C.

Figure 11.9

	C*				
	20.00	30.00	40.00	50.00	60.00
σ_P					
0.00	56.51	64.84	71.01	76.52	81.88
0.10	55.46	63.31	69.02	74.16	79.20
0.20	53.47	59.98	64.44	68.59	72.80
0.30	51.64	56.44	59.28	62.11	65.23
0.40	50.22	53.34	54.49	55.96	57.96
0.50	49.15	50.79	50.36	50.54	51.51

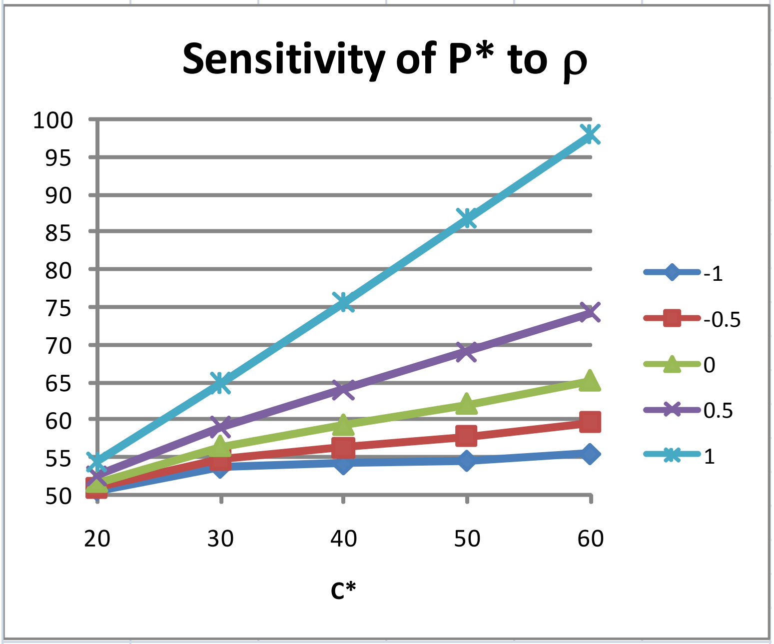


	SPREAD P*-C*				
σ_P					
0.00	36.51	34.84	31.01	26.52	21.88
0.10	35.46	33.31	29.02	24.16	19.20
0.20	33.47	29.98	24.44	18.59	12.80
0.30	31.64	26.44	19.28	12.11	5.23
0.40	30.22	23.34	14.49	5.96	-2.04
0.50	29.15	20.79	10.36	0.54	-8.49

The effects of various correlation coefficient values on the renewal boundary are illustrated in Figure 11.10. The boundaries for $-1 \leq \rho \leq 1$ are not exactly linear and have varying positive slopes. The boundary slope attains its greatest value for perfect positive correlation but is nearly zero for $\rho = -1$. Prevailing operating costs hardly affect the renewal decision for $\rho = -1$. This is explained by the nature of the threshold levels, which collectively represent the positive trade-off between the two variables along the renewal boundary. If the prevailing sales and operating

Figure 11.10

	C*				
	20.00	30.00	40.00	50.00	60.00
ρ					
-1.00	50.61	53.63	54.10	54.57	55.49
-0.50	51.05	54.81	56.27	57.73	59.56
0.00	51.64	56.44	59.28	62.11	65.23
0.50	52.56	59.02	64.05	69.09	74.31
1.00	54.43	64.89	75.61	86.72	97.89



	SPREAD P*-C*				
ρ					
-1.00	30.61	23.63	14.10	4.57	-4.51
-0.50	31.05	24.81	16.27	7.73	-0.44
0.00	31.64	26.44	19.28	12.11	5.23
0.50	32.56	29.02	24.05	19.09	14.31
1.00	34.43	34.89	35.61	36.72	37.89

cost fall on the boundary, the outcome of a sales increase (decrease) accompanied simultaneously with an operating cost increase (decrease) can belong to either the continuance or renewal region. In contrast, when a sales increase (decrease) accompanies an operating cost decrease (increase), the outcome always belongs to the continuance (renewal) region. When this occurs, the decision to continue or renew depends on only the prevailing sales level. If the correlation is perfectly negative and the variances are equal, a random shock will increase (decrease) sales and decrease (increase) operating costs by an identical amount. It follows that the renewal decision is governed by only the sales level. In contrast, the renewal boundary has a significantly positive slope for $\rho > 0$ and the prevailing values of both variables are relevant. In summary, sales and operating costs cannot be legitimately assimilated into a single variable for the normal case of zero or positive correlation. For the rare case where the two variables are perfectly negatively correlated, the renewal decision can be approximately decided by the prevailing sales level.

Another interpretation is that the subsequently repeated spread between expected pairs of $\hat{P}$ and $\hat{C}$ that justifies immediate renewal increases with increases in correlation, and declines with increases in $\hat{C}$.

While the effect of cost volatility, sales and cost drifts and interest rates should be considered, the sales reversion level, expected sales volatility, and sales and cost correlation are the most important drivers of $\hat{P}$ and $\hat{C}$.

SUMMARY

Multiple asset renewals are appropriate for entities that have a plausible continual existence forever (except in cases of extreme fashion or product downgrading creating a single renewal possibility, or low reversionary P and high C justifying abandonment), and yet equipment quality and effectiveness (P) and efficiency (C) deteriorate with time and/or usage.

There are several basic models in this chapter: multiple two factor renewals, deterministic and one-factor renewals, and two factor single renewals and abandonments. The deterministic equations are (11.23), (11.24) and (11.25). The one-factor equations are (11.26) and (11.27), where the sales trigger is of interest (assuming cost is constant). The multiple two-factor equations are (11.4), (11.11) and (11.13). All of these equations are solved simultaneously, assuming an identified cost trigger level. Using the same parameter values, the deterministic model shows where $\hat{C}$=29.9 the optimal deterministic $\hat{P}$ =65.4, the one-factor $\hat{P}$ =51.60, the two-factor $\hat{P}$ =57.08, with a specific deterministic technological progress $\hat{P}$=58.07. If the current P=60, renewal is justified using the deterministic model but a long wait is indicated using the one-factor model (which ignores the 4% annual increase in cost of the other models). There are some simplifications possible even for the multiple stochastic P and C model as shown in Figure 11.4, which are, however, parameter specific. For instance, for $\hat{C}$>30, $\hat{P}$=47+.3$\hat{C}$ is a reasonable approximation. For the deterministic case, a year by year comparison of sales and costs deterioration is easy to construct.

Certain parameters are important drivers of justifiable multiple renewals, P volatility as shown in Figure 11.9, and correlation as shown in Figure 11.10. Practical implications for CFOs and equipment managers are that if expected sales volatility is high, don’t worry about cost levels, but defer renewals until current sales are low. In order to encourage sales, equipment saleswomen should emphasize low future sales volatility. Similarly if sales and cost correlation is negative, cost levels don’t matter; saleswomen should search for possible high correlation on sales and costs, and above all high sales produced by new equipment.

Focused managers will discover the critical drivers of their particular asset renewal case, and some may want to explore more renewal models which concern fashion and technical innovation jumps, salvage or second hand values, competition and taxation.

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EXERCISES

EXERCISE 11.1

Bobby Riskins thought he needed a new car, since his seven year old Buick had maintenance costs increasing by 10% per annum from the initial $$20; he could not stand costs over $$40.1=C*. He didn't think much of the old Buick style, equal to a car enjoyment deterioration rate of 10% per annum. A new car costs $$150 and would bring his enjoyment level up to the original $$80. You are a car salesman, and almost honest, so advise Bobby using generally accepted NPV methods with r=14%. HINT: Figure T_C* using EQ 11.23. For P* try EXP(ln(P_I)+ T_C*θ_P) as an approximation. What is P_0*EXP (θ_{P} *7)?

EXERCISE 11.2

Tour de France dreamer, Lens Footstrong, believes he could go faster on a new cycle, since race times have been shown to be faster each year for new cycles. New cycles cost a fortune, 100, and Lens is not convinced that he wants to pay that much to go faster. 100% speed is worth 80, constant exercise to obtain this level is 30. Using his current cycle, he is now at half speed and losing in races. He knows that β=-1, due to the current interest rates (14%), speed volatility (30%) and his belief that a new cycle speed rate would deteriorate at a rate of 5% per annum. Advise him. HINT: P* is the solution to a quadratic equation, where a=(1/(-(r-θ)P_I), b=(β-1)/-(r-θ), c=-((P_I/(r-θ))-K), see Chapter 4, Appendix.

EXERCISE 11.3

Michael Funagan, owning Funair initially operating out of Tullamore, was thinking of replacing his ten year old airplane. He knew that with a new plane P=80, K=100, interest rates are 8%, sales have a 20% volatility and decline by 4% per annum. His low costs start at $$1 but increase by 2% p.a. Now sales are around 50. Should he wait or renew? HINT: both β and P* are solutions to quadratic equations, where for P*, a=(1/(-(r-θ)P_I), b=(β-1)/-(r-θ), c=-((P_I/(r-θ))-K).

PROBLEMS

PROBLEM 11.4

Bobby's mom was the star of her real options class, and knew that his pleasure and car costs were highly volatile (40%) and completely negatively correlated. "Wait, my boy!" Is she right? How much longer must Bobby endure his mom's cleverness, before getting a shiny new red car to impress the girls?

PROBLEM 11.5

Another Tour de France dreamer, Daniel Darcy, believes he could go faster on a new cycle, since race times have been shown to be faster each year for new cycles. New cycles cost a fortune, 100, and Daniel is not convinced that he wants to pay that much to go faster. 100% speed is worth 80, constant exercise to obtain this level is 30. Using his current cycle, he is now at half speed, and will hardly qualify for the Tour. He knows that $\beta=-2$, due to the current interest rates (14%), speed volatility (11.547%) and his belief that a new cycle speed rate would deteriorate at a rate of 5% per annum. Advise him.

PROBLEM 11.6

Michael Funagan, owner of Funair now operating out of Dublin and Manchester, was thinking of replacing his fleet of ten year old airplanes. He knew that with a new plane P=80, compared to current sales of 60, K=100, interest rates are 7%, net revenue have a 30% volatility and decline by 4% per annum. His low costs were originally 20 but are now around 30, (with a 30% volatility), perfectly negatively correlated with sales under a profit sharing plan with employees, but increasing by 4% every year. A new MBA from the Summit College of Business understood Excel, so approximations were not required. Should Michael wait or renew?